The Heart of Modern Probability Theory

It is truth very certain that, when it is not in our power to determine what is true, we ought to follow what is most probable.

-René Descartes1

Scientists cannot say how the world came to be or how life began. There was no human observer on the scene to record with technical data whether God created or things just evolved. Until one arrives at faith in the Bible record, the only logical course is to use inductive reasoning and follow what is most probable. Let us begin now to learn the main rules of probability. We will need only two central principles. Here is the first, sometimes called the "law of averages."

The Law of Large Numbers

Probability theory applies mainly to "long runs." If you toss a coin just a few times, the results may vary a lot from the average. As you continue the experiment, however, it levels out to almost absolute predictability. This is called the "law of large numbers." Here is how physicist George Gamow stated it:

Thus whereas for 2 or 3, or even 4 tosses, the chances to have heads each time or tails each time are still quite appreciable, in 10 tosses even 90 per cent of heads or tails is very improbable. For a still larger number of tosses, say 100 or 1000, the probability curve becomes as sharp as a needle,

¹ In Darrel Huff and Irving Geis, *How To Take a Chance* (New York: W. W. Norton & Co., 1959), p. 7.

and the chances of getting even a small deviation from fifty-fifty distribution becomes practically nil.²

The long run serves to average out the fluctuations that you may get in a short series. These variations are "swamped" by the long-haul average. When a large number of tries is involved, the law of averages can be depended upon quite closely. This rule, once called the "law of great numbers," is of central importance in this field of probability. By the way, in the popular sense, probability theory, the laws of chance, and the science of probability can be considered to be simply different expressions for the same general subject.

Make Your Experiments Scientific

To be exact, the theory of probability deals not with material objects, but with *ideal theoretical models* or mental pictures. If we use objects that are reasonably identical, however, the results of our experiments will be close to the same as with the abstract mathematical models on which the laws are based.

When we do experiments such as coin tosses or drawings of numbered objects, it is important to insure that there is equal likelihood of the different outcomes or "events" as they are called. If one of the objects to be drawn is heavier than the others, it may tend to settle to the bottom of the group, thus giving results that are inaccurate. Different rules might be involved if the various possible results are not made equally probable.

In selecting coins or letters at random, they must, of course, be thoroughly mixed before each drawing. If they are not shaken sufficiently, the same one that was just drawn might remain near the top to be more easily drawn again. Objects also should be drawn without looking, to avoid the possibility that the choice is influenced by sight of the various objects. The purpose is to find out what chance can do, and chance is blind.

If other articles are used instead of coins, they should as nearly as possible be the same size and shape and weight. This makes the experiment more scientific and assures more accurate results. Experimenting may mean more to you if first you read on a few pages farther.

The Multiplication Rule (Learn It Well!)

We now come to the most important rule of all for the

² George Gamow, One, Two, Three-Infinity (New York: Viking Press, 1961), p. 209.

purposes of our study. It is the second of the two principles. Let's go back to the ten numbered coins. Why is there only one chance in one hundred that we will get the number one coin on the first draw followed by the number two coin on the next draw?

Here is the principle involved, as described clearly by Adler: "Break the experiment down into a sequence of small steps. Count the number of possible outcomes of each step. Then multiply these numbers." This important "multiplication rule" is most often used where the various outcomes of a particular step are all equally probable and the steps are independent.

In the experiment with ten similar coins numbered one through ten, we want to know the probability of getting the number one coin on the first try followed by the number two coin on the second try. Divide this into steps as Adler suggested. Our first step will be to draw one coin. There are ten different outcomes we could get on that first draw. There are also ten different possible results when we get to the second step. Multiplying, as Adler said, we have $10 \times 10 = 100$. So, the chance is 1 in 100 of getting the two desired coins in order. The probability is 1/100, on the average.

Before the first draw, we know intuitively that there is a 1-out-of-10 chance of success in getting the number one coin.4 Therefore, whatever chance the second step will have must be multiplied by 1/10, because there is only that 1/10 chance of success on the first step. But the second step also has 1/10 probability of success. As we have just seen, that will have to be multiplied by the 1/10 probability from step one. This will give the answer for both steps together, which is 1/100. If such an experiment is continued long enough, about once in every hundred draws the number one coin will be followed by the number two. Remember, however, the law of large numbers. There will be deviations unless you do several hundred and average them.

The principle is: If you seek first "this outcome" and then "that outcome," the probability of getting both is the product of their separate probabilities, in cases where one outcome does not affect the other. George Gamow said it in these words:

³ Irving Adler, *Probability and Statistics for Everyman* (New York: John Day Co., 1963), pp. 58, 59.
⁴ This probability arises partly because of equivalence or symmetry, and we

sense its logic.

Here we have the rule of "multiplication of probabilities," which states that if you want several different things, you may determine the mathematical probability of getting them by multiplying the mathematical probabilities of getting the several individual ones,⁵

Perhaps this may seem to be much ado about a minor point. Some who are mathematically minded or knew the principle beforehand may have gotten it easily. For most people, however, it is hard to believe that the chances are that slim—just one in one hundred. This is the average outcome one can expect.

It will be worthwhile to stay with this matter until thoroughly convinced that it is true. One's mind may be slow to accept the idea. Darrell Huff wrote that "even intelligent adults confuse addition with multiplication of probabilities." That is why actual experimenting may be such a help. Much depends on becoming certain in one's own thinking that this is correct. A little later, we will suggest quicker methods for experimenting that will lead to the certainty of the truth of this rule.

This one point is absolutely vital to the whole process of this approach to certainty. It may be mastered by rereading and by experimenting as described a little farther on, and by pondering the matter until one's mind will accept its truth. All probability theory used in science and industry builds from this multiplication rule.

Can Chance Count to Ten?

To absorb the meaning of that fully is to be well on the way to the assurance that we seek. Chance requires ten billion tries on the average in order to count to ten!

Shorten Your Experiment Time

The reader has doubtless already realized that the experiment

⁵ Gamow, One, Two, Three-Infinity, p. 208.

with ten coins is too long for any reasonable chance of success if done properly. If a person could draw and record one coin every five seconds day and night, it would take over 1,500 years to complete the time in which *one* success could be expected! In all that time, the outlook is for chance, on the average, to succeed just once in counting to ten.

Perhaps we get the gist of the idea that chance is not very capable when we need an ordered result. Consider the difference intelligence makes—even a limited intelligence. Give an eight-year-old the coins, and ask the child to arrange and pick up each one in order and return it. Chance is blind, and has no intelligence. The child is not thus limited. The child can do it in a few moments. Chance takes 1,500 years—just to count to ten once.

The same principle can be learned with shorter experiments, using fewer coins. If you try it with three or four or five numbered coins long enough to average out any short-run fluctuations, you will see that the rules hold true. With five coins, the probability of getting the number one and the number two in order on the first two draws is naturally 1 in $5 \times 5 = 1$ in 25.

In tossing a coin, the probability of four heads in a row is $1/2 \times 1/2 \times 1/2 \times 1/2 = 1/16$. What would be the probability of ten heads in a row?

To Spell "Evolution" by Chance

Suppose, instead of numbers, we use the letters of the alphabet. As a substitute for coins, any small, similarly shaped objects may be used if they are practically *identical* in size, weight, and shape. (The party game called "Scrabble" has small letters on wooden squares quite suitable for this.)

With one set of the twenty-six letters of the alphabet, you have 1/26 probability of getting the "A" on the first draw. To get "A" followed by "B" (replacing the letter after each draw, as before) your probability by the multiplication rule is: $1/26 \times 1/26 = 1/676$. To get ABC in order, the chance is 1 in 17,576, by the same rule.

To spell the word "evolution," obtaining the nine letters in order, each having a 1/26 probability, you have a probability

⁶ We tried such an experiment at the Center for Probability Research in Biology. In 30,000 alphabet letters drawn, only once did we get ABC in order! (Of course, there were other reasons for the experiment. The main purpose is explained in chapter 6 where it provides an analogy for usable and nonsense chains of amino acids.)

of 1 in 5,429,503,678,976. This, as you will realize, comes from multiplying 26 by itself, using the figure 9 times. If every five seconds day and night a person drew out one letter, he could expect to succeed in spelling the word "evolution" about once in 800,000 years!

Further Tests for Chance

Suppose we put chance to a test which is less simple, yet something that would be quite easy for any school child. Let it spell this phrase: "the theory of evolution." Drawing from a set of twenty-six small letters and one blank for the space between letters, what is the probability expectance?

All that is needed is simply to get those twenty-three letters and spaces in proper order, selecting them at random from the set of twenty-seven objects (twenty-six letters and one space). By the multiplication rule we learned, it will be $27 \times 27 \times 27 \dots \times 27$ using the figure twenty-three times.

The probability when computed is 1 in approximately 834,-390,000,000,000,000,000,000,000,000,000; that is, one success in over 8 hundred million trillion trillion draws.

To get an idea of the size of that number, let us imagine that chance is employing an imaginary machine which will draw, record, and replace the letters at the speed of light, a BILLION draws PER SECOND! Working at that unbelievable rate, chance could spell "the theory of evolution" once in something over 26,000,000,000,000,000 years on the average!

Again, a child could do it in a few minutes. Chance would take more than five million times as long as the earth has existed (if we use the five-billion-year rounded figure which some evolutionists now estimate as the age of the earth).

Chance Is Moronic

So chance requires twenty-eight trillion times the age of the earth to write merely the phrase: "The Theory of Evolution," drawing from a set of small letters and capitals as described,

drawing at the speed of light, a billion draws per second! Only once in that time could the letters be expected in proper order.

Again, a child can do this, using sight and intelligence, in a few minutes at most. Mind makes the difference in the two methods. Chance really "doesn't have a chance" when compared with the intelligent purpose of even a child.

"In the beginning, God..." begins to appear more scientific, as we see how limited are the abilities of mindless chance.

Perhaps the alphabet experiments just described may help to emphasize how important it is fully to understand the multiplication rule we studied earlier. It's hard to believe at first. Try drawing alphabet letters for a few hours to become really convinced! Remember in doing so that chance has no intelligence, no purpose. It cannot purposefully choose one correct letter and discard unwanted ones until it finds the next one needed.

In the next two chapters, we will make some actual use of what we have learned. We are to apply probability theory to the strange phenomenon of the "left-handed" molecules which are used in proteins. We will use that as a practice field in applying the laws of chance. It is ideal for this, because only two possible outcomes are involved for each step. It is similar, therefore, to the experiment of tossing a coin.

Special Note to the Reader

Most of this book is in plain, easy-to-understand language. In a few places, however, we must go far enough into certain areas of biology to apply the laws of chance in logical manner. This will require the use of a small amount of mathematics, but not much—mostly just arithmetic. It is a necessary part of the process in gaining certainty by the approach which we are following.

For the reader who happens to have an absorbing interest in biology, it is unlikely to involve any strain or confusion as a rule.

Perhaps, on the other hand, you have only a casual interest in the details of science. Does that rule out the value to you of this method of seeking assurance on evolution? Not at all. A great number of people may not have any engrossing interest in biology, and yet may attain that valuable certainty.

⁷ The imaginary machine is considered as moving slightly less than a foot per draw, round trip. The letter is recorded during the return trip so that no time is taken up except the actual travel, round trip, of .98 foot at the speed of light, to allow 1,000,000,000 draws per second.

If you plow on through any places that seem somewhat technical, you will at least get the general idea and you will soon be back into easier reading. In the process, you will realize that the actual facts and figures are there in print for anyone who wishes to dig into the subject more thoroughly. The conclusions, moreover, are always in easily grasped speech. Without the actual reasoning and figures, and without the references, the reader would have little to depend on except an author's words, and that is a poor basis for certainty. Don't worry, then, if you strike sections that you do not quickly comprehend completely. Just read on through. You can return later to those sections that you may wish to reread.

Before going on, we will confess that (to the horror of mathematicians) we have oversimplified a bit, to make the ideas accessible to people not trained in mathematics. The recurrent phrase, "on the average," needs more explaining when it is used with experiments which are repeated. The footnote below goes into this, for the noncasual reader.⁵ Now, let's look at left-handed molecules.

⁸ Our figuring thus far has been the kind where "success" was getting a certain result *once*, *on the average* in a series of trials. Now, consider the different concept of *at least once*: the desired event may happen once or more than once, but the main thing is that it happens at all.

If we draw from ten coins (with replacement), what is the probability that the No. 1 coin will show up at least once if we make two draws? Here is what can happen: (1) We may obtain the No. 1 coin just once from the two draws; (2) we may get it both times; or (3) not at all. Either the first or the second of these results would be a "success," because in each the event occurs: "No. 1 coin at least once."

We see that success can happen in more than one way, but failure can happen just one way. We therefore first figure the chance of failure. It is 9/10 on any one draw, and we can use the multiplication rule for two draws, because we need failure both times—"this and that." $9/10 \times 9/10 = 81/100$. Now to find the chance of success:

Always, if one adds the probability of success and the probability of failure, the total is exactly one. We can obtain the probability of success by subtracting 81/100 from 100/100 (which is the same as one). The answer is 19/100, the chance of getting the No. 1 coin at least once. A mathematician might write the formula thus: where n is the number of draws, and p is the probability of success in one draw: $p_n = 1 - (1 - p)^n$.

With the large figures we will encounter, it would make virtually no difference

With the large figures we will encounter, it would make virtually no difference if we used this more exact method, so we will save confusion by figuring the much simpler probability on the average. Chapter 10 will give more details on this. (The difference between the two methods is less than just adding one to an exponent of ten. The exact method would be even harder on evolution.)